MATHEMATICS EDUCATION AND APPLIED LINGUISTICS: WORKING GROUP REPORT

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Continuing one of the themes initiated in earlier meetings, the working group considered the question of what linguistic analytic tools can offer to research in mathematics education. In this session we focused on the place of definitions within mathematical practices. Using extracts taken from a range of mathematical texts, Candia Morgan offered critical discourse analyses of definitions, using grammatical tools drawn from functional linguistics, as a starting point for discussion of issues such as: the relationship between a mathematical object or concept and its definition; roles played by definitions in the practices of doing mathematics; what analysis of definitions presented to learners at different stages can contribute to our understanding of these issues.

INTRODUCTION

The starting point for this topic was the conflict identified between the strong emphasis given to "technical mathematical vocabulary" by the National Numeracy Strategy (DfES, 2000) and observation of the ways in which mathematical concepts are actually talked about in a primary classroom. This was discussed in the first meeting of this working group (Barwell, Leung, Morgan, & Street, 2002b). In particular, the NNS advice that teachers should explain the meanings of new words carefully and "sort out any ambiguities and misconceptions" (DfES, 2000, p.2) suggests a one-to-one relationship between word and concept that can be embodied in a definition, while study of a transcript of a class discussion of *dimension* showed that the teacher and children were using multiple definitions of the word – all legitimate (Barwell, Leung, Morgan, & Street, 2002a). In this session, we looked at some examples of definitions as they are presented to students at various stages of learning. Candia Morgan introduced some analytic tools drawn from functional grammar (Halliday, 1985) and showed how they might be applied to illuminate the ways in which the nature of mathematics and mathematical activity may be constructed through the texts presented to learners (Morgan, 1996). The group then discussed issues raised by this analysis and other examples of definitions encountered in different kinds of texts.

ANALYTIC TOOLS

Descriptions of key features of the definition texts were identified using the following questions and associated grammatical tools:

Descriptive questions:	Grammatical tools:
1. Who or what are the actors and where does agency lie?	What objects and humans are present? How are active or passive voice used?
2. What are the processes?	Relational, material, mental/behavioural?
3. Describe the modality.	Modal verbs, adverbs, adjectives
4. How is the status of 'definition' established textually?	Given/New structures; how cohesion is achieved.

The first two questions are related to Halliday's (1973) ideational function of language, question 3 to the interpersonal function and question 4 to the textual function.

The grammatical description then allows us to address critical questions about how the text may contribute to student-readers' positioning in relation to mathematics and mathematical activity, asking in particular: *What is the nature of mathematics/ mathematical objects/ mathematical activity?* and *Where do power and authority lie?*

EXAMPLE ANALYSES

The two examples chosen as a starting point were taken from textbooks in the same series, written by the same authors, intended for students in Key Stage 4 preparing for GCSE examinations at Intermediate and at Higher level. They both present definitions of trigonometric concepts, though at different levels.

Example 1: GCSE Intermediate Textbook

In Investigation 15:1, you found that the ratio $\frac{\text{shortest side}}{\text{longest side}}$ i.e. $\frac{\text{opposite}}{\text{hypotenuse}}$ is the same for each of these triangles.

This ratio is given a special name. It is called the sine of 40° or **sine 40°**.

The ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ is called **cosine 40°**. The ratio $\frac{\text{opposite}}{\text{adjacent}}$ is called **tangent 40°**.

The abbreviations sin, cos, tan are used for sine, cosine, tangent.

The ratios sin A, cos A, tan A are called **trigonometrical ratios**, or **trig. Ratios**.

Example 2: GCSE Higher Textbook

The ratios $\sin\theta$ and $\cos\theta$ may be defined in relation to the lengths of the sides of a right-angled triangle.

$\sin\theta$ is defined as $\frac{\text{length of opposite side}}{\text{length of hypotenuse}}$

$\cos\theta$ is defined as $\frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$.

Since $\theta < 90$, $\sin\theta$ and $\cos\theta$ defined in this way only have meaning for angles less than 90°.

> We will now look at an alternative definition for $\sin\theta$ and $\cos\theta$ which has meaning for angles of any size. (…) This gives the following alternative definition for the ratios $\cos\theta$ and $\sin\theta$.
>
> The ratios $\cos\theta$ and $\sin\theta$ may be defined as the coordinates of a point P where OP makes an angle of θ with the positive x-axis and is of length 1. Defined in this way, the ratios $\cos\theta$ and $\sin\theta$ have meaning for angles of any size.

An analysis using grammatical tools is given in the table on the next page, laid out to facilitate comparison of the two texts.

Working group participants identified some significant differences between the two texts that are apparent from this grammatical description. Considering first of all the nature of mathematics and mathematical activity in the context of definition, in both texts the agency in the act of naming or defining is obscured by use of the passive voice but the types of activity in which human actors are agents are different. In the Intermediate text, the student herself is presented as having been involved in an earlier practical activity. In the Higher text, there is no practical activity but 'we' are engaged in the intellectual activity of looking at an alternative definition. The forms of the two texts themselves also contribute to differences in the type of activity that is constructed as mathematical. The Intermediate text is essentially descriptive, starting with what is known about a specific concrete example and extending the description to naming a more general set of similar objects. The object/concept of the ratio between two sides of a triangle is established as the outcome of practical activity before it is named. This order is reversed in the Higher text: the choice of an alternative definition changes the nature of the object being defined. This text also uses structures that highlight the formation of a logical argument – an aspect of mathematical activity absent from the Intermediate extract.

The second major difference arises from the modality of the two texts. While the Intermediate text lays down a set of absolute and unquestionable facts to be accepted by the student-reader, the Higher text allows uncertainty and alternatives, opening up the possibility that the student-reader herself might choose between the two definitions. The student entered for the Higher level examination is thus constructed as a potential initiate into the practices of creative and purposeful definition that academic mathematicians engage in.[1]

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[1] Similar differential access to mathematical practices is identified by Paul Dowling in his analysis of a differentiated textbook scheme (Dowling, 1998). In that case, the 'lower' students were constructed as engaged in 'everyday' practices and were denied access to esoteric mathematical practices.

	GCSE Intermediate	GCSE Higher
Actors & Agency	• 'You found …' – student agent in practical activity • The ratio 'is given a special name' 'is called' and 'the abbreviations … are used' – passive voice obscures agency	• 'The ratios … may be defined' – passive voice • Sin and cos 'have meaning' • 'We will look' – is this the authors or is it an inclusive 'we'? In either case, there is some human agency here.
Processes	• Material process 'found' by student • Behavioural processes 'call', 'use' which would normally require a sentient agent but here are in the passive voice	• Behavioural processes 'define' and 'look' • Relational (intensive) 'have [meaning]'
Modality	• Generally neutral i.e. absolute modality (these are given facts – no questions asked) • The ratio is given 'a special name' – stressing the importance of the new vocabulary	• Modification of verbs to reduce level of certainty – 'may be defined'. This opens up the possibility of alternative ways of doing things - and the possibility that the student might be able to make choices. • Similar adverbial and adjectival modifications: 'defined in this way', 'an alternative definition'
Textual status of 'definition'	• All sentences except the first have unmarked word order: the ratio (found by the student) is the given knowledge; the mathematical terminology is the new. • Move from a specific example of a concrete object (the ratio of opposite to hypotenuse in a 40° triangle) to giving a name to this object and to extending this naming to general similar objects – thus the object/concept pre-exists the naming of it • Cohesion achieved by repetition of 'the ratio' and its cognates in the thematic position, presenting the text as a collection of facts about 'the ratio' – description.	• In the final sentence, word order is marked by positioning the adverbial phrase 'defined in this way' in the 'given knowledge' position. The form of the definition is presented as changing the meaning of the object – thus definition precedes object/concept • 'Since $\theta < 90$' in a thematic position presents the text as a process of logical argument.

DISCUSSION

As well as looking at these GCSE level examples in some detail, the working group was able to make comparisons with definitions taken from texts aimed at students at Key Stage 3, A-level and undergraduate level as well as definitions used in a mathematical research paper. These examples were not selected in any systematic way so it would not be appropriate to draw conclusions about differences between various types of texts. The analysis does, however, raise some questions and hypotheses about the ways that definitions are presented to students at different levels. The tools used in the analysis allow a systematic way of identifying and describing such differences.

An important point raised in the discussion was related to the ways in which textbooks are actually used. In most classrooms, the text is likely to be mediated by the teacher and this will affect the ways in which students interact with the text themselves. As students construct their understandings of the nature of mathematics and mathematical activity and of their own identities in relation to mathematics they will draw to different extents on the textbook, the teacher's speech and actions and on their previous experiences. However, it was also pointed out that, where teachers are insecure in their own subject knowledge they are likely to rely heavily on the forms of definition and argumentation that are provided for them in published resources.

REFERENCES

Barwell, R., Leung, C., Morgan, C., & Street, B. (2002a). The language dimension of maths teaching. *Mathematics Teaching, 180*, 12-15.

Barwell, R., Leung, C., Morgan, C., & Street, B. (2002b). Mathematics education and applied linguistics: Working group report. *Proceedings of the British Society for Research into Learning Mathematics, 22*(2), 89-94.

DfES. (2000). *The National Numeracy Strategy: Mathematical Vocabulary*. London: Department for Education and Skills.

Dowling, P. (1998). *The Sociology of Mathematics Education: Mathematical Myths/ Pedagogic Texts*. London: Falmer.

Halliday, M. A. K. (1973). *Explorations in the Functions of Language*. London: Edward Arnold.

Halliday, M. A. K. (1985). *An Introduction to Functional Grammar*. London: Edward Arnold.

Morgan, C. (1996). "The language of mathematics": towards a critical analysis of mathematical text. *For the Learning of Mathematics, 16*(3), 2-10 + erratum *17*(11).